2. Základy lineárnej algebry

2.1. štvorcová: B,D,F,G,H
nulová: I
diagonálna: G, H
jednotková: G
symetrická: G,H
trojuholníková: A,D,E,F,G,H

2.2.

$$A^T=\begin{pmatrix}-1&0\\2&2\\3&0\end{pmatrix};\quad B^T=\begin{pmatrix}-1&2&2\\2&2&0\\3&0&5\end{pmatrix};\quad C^T=\begin{pmatrix}-1&0&1&1\\2&2&3&0\end{pmatrix};\quad D^T=\begin{pmatrix}4&0&0&0\\0&2&0&0\\1&0&-9&0\\3&1&1&5\end{pmatrix};\quad E^T=\begin{pmatrix}-2&0\end{pmatrix}$$

$$F^T=\begin{pmatrix}-1&0\\3&2\end{pmatrix};\quad G^T=\begin{pmatrix}1&0\\0&1\end{pmatrix};\quad H^T=\begin{pmatrix}4&0&0\\0&2&0\\0&0&-5\end{pmatrix};\quad I^T=\begin{pmatrix}0\\0\end{pmatrix}$$

$$B+H=\begin{pmatrix}3&2&3\\2&4&0\\2&0&0\end{pmatrix}_{3x3};\quad F+G=\begin{pmatrix}0&3\\0&3\end{pmatrix}_{2x2}$$

$$A.B=\begin{pmatrix}11&2&12\\4&4&0\end{pmatrix}_{2x3};\quad A.H=\begin{pmatrix}-4&4&-15\\0&4&0\end{pmatrix}_{2x3};\quad B.H=\begin{pmatrix}-4&4&-15\\8&4&0\\8&0&-25\end{pmatrix}_{3x3}$$

$$C.A=\begin{pmatrix}1&2&-3\\0&4&0\\-1&8&3\\-1&2&3\end{pmatrix}_{4x3};\quad C.E=\begin{pmatrix}2\\0\\-2\\-2\end{pmatrix}_{4x1};\quad C.F=\begin{pmatrix}1&1\\0&4\\-1&9\\-1&3\end{pmatrix}_{4x2}$$

$$C.G=\begin{pmatrix}-1&2\\0&2\\1&3\\1&0\end{pmatrix}_{4x2};\quad D.C=\begin{pmatrix}0&11\\1&4\\-8&-18\\5&4\end{pmatrix}_{4x2};\quad E.I=\begin{pmatrix}0&0\\0&0\end{pmatrix}_{2x2}$$

$$F.A=\begin{pmatrix}1&4&-3\\0&4&0\end{pmatrix}_{2x3};\quad F.E=\begin{pmatrix}2\\0\end{pmatrix}_{2x1};\quad F.G=\begin{pmatrix}-1&3\\0&2\end{pmatrix}_{2x2}$$

$$G.A=\begin{pmatrix}-1&2&3\\0&2&0\end{pmatrix}_{2x3};\quad G.E=\begin{pmatrix}-2\\0\end{pmatrix}_{2x1};\quad H.B=\begin{pmatrix}-4&8&12\\4&4&0\\-10&0&-25\end{pmatrix}_{3x3}$$

$$I.A=\begin{pmatrix}0&0&0\end{pmatrix}_{1x3};\quad I.E=\begin{pmatrix}0\end{pmatrix}_{1x1};\quad I.F=\begin{pmatrix}0&0\end{pmatrix}_{1x2};\quad I.G=\begin{pmatrix}0&0\end{pmatrix}_{1x2}$$

2.3. a) $x=\frac{-2}{5}$, $y=1$ b) $x=\frac{1}{4}$, $y=\frac{1}{2}$, z= $\frac{5}{3}$, u= 2

2.4.

a) $A=\begin{pmatrix} 2 & -5 & -1 \\ -3 & 2 & -20 \end{pmatrix}$ b) $B=\begin{pmatrix} 2 & 4 & -8 \\ -5 & -2 & -3 \\ 5 & 7 & 26 \\ 9 & -16 & 11 \end{pmatrix}$

2.5.

a) $X=\begin{pmatrix} -1 & 15 & -2 \\ 7 & 0 & -4 \\ -3 & 7 & 3 \end{pmatrix}$ b) $X=\begin{pmatrix} -5 & -5 & -6 & -4 \\ -2 & 0 & -22 & 0 \\ 3 & 1 & -10 & -17 \end{pmatrix}$

2.6.

$A_{12}=\begin{pmatrix} 8 & 0 & -2 \\ 2 & -9 & 1 \\ 0 & 0 & 5 \end{pmatrix}$; $A_{22}=\begin{pmatrix} 4 & 1 & 5 \\ 2 & -9 & 1 \\ 0 & 0 & 5 \end{pmatrix}$; $A_{31}=\begin{pmatrix} -1 & 1 & 5 \\ 2 & 0 & -2 \\ -5 & 0 & 5 \end{pmatrix}$; $A_{14}=\begin{pmatrix} 8 & 2 & 0 \\ 2 & 0 & -9 \\ 0 & -5 & 0 \end{pmatrix}$

$A_{11}\cdot A_{22}=\begin{pmatrix} 32 & 8 & 30 \\ -10 & 83 & 6 \\ 0 & 0 & 25 \end{pmatrix}$; $A_{31}\cdot A_{14}=\begin{pmatrix} -6 & -27 & -9 \\ 16 & 14 & 0 \\ -40 & -35 & 0 \end{pmatrix}$; $A_{12}\cdot A_{12}=\begin{pmatrix} 64 & 0 & -26 \\ -2 & 81 & -8 \\ 0 & 0 & 25 \end{pmatrix}$

$A_{22}\cdot A_{31}=\begin{pmatrix} -27 & 4 & 43 \\ -25 & 2 & 33 \\ -25 & 0 & 25 \end{pmatrix}$; $A_{22}\cdot A_{22}=\begin{pmatrix} 18 & -5 & 46 \\ -10 & 83 & 6 \\ 0 & 0 & 25 \end{pmatrix}$

2.7.

a) $\det A_{12}=-360$; $\det A_{22}=-190$; $\det A_{31}=0$; $\det A_{14}=360$

b) $\det(A_{12}\cdot A_{22})=68400$; $\det(A_{31}\cdot A_{14})=0$; $\det(A_{22}\cdot A_{31})=0$

c) $\det A_{12}^T=-360$; $\det A_{22}^T=-190$; $\det A_{31}^T=0$; $\det A_{14}^T=360$

d), e) dosadíte

2.8.

a) $\det A=(-1)^{1+1}\cdot 4\cdot\begin{vmatrix} 2 & 0 & -2 \\ 0 & -9 & 1 \\ -5 & 0 & 5 \end{vmatrix}+(-1)^{1+2}\cdot(-1)\cdot\begin{vmatrix} 8 & 0 & -2 \\ 2 & -9 & 1 \\ 0 & 0 & 5 \end{vmatrix}+(-1)^{1+3}\cdot 1\cdot\begin{vmatrix} 8 & 2 & -2 \\ 2 & 0 & 1 \\ 0 & -5 & 5 \end{vmatrix}+(-1)^{1+4}\cdot 5\begin{vmatrix} 8 & 2 & 0 \\ 2 & 0 & -9 \\ 0 & -5 & 0 \end{vmatrix}=$

$=1480$

b) $\det A=(-1)^{3+1}2\cdot\begin{vmatrix} -1 & 1 & 5 \\ 2 & 0 & -2 \\ -5 & 0 & 5 \end{vmatrix}+(-1)^{3+2}\cdot 0\cdot\begin{vmatrix} 4 & 1 & 5 \\ 8 & 0 & -2 \\ 0 & 0 & 5 \end{vmatrix}+(-1)^{3+3}(-9)\begin{vmatrix} 4 & -1 & 5 \\ 8 & 2 & -2 \\ 0 & -5 & 5 \end{vmatrix}+(-1)^{3+4}\cdot 1\begin{vmatrix} 4 & 1 & 1 \\ 8 & 2 & 0 \\ 0 & -5 & 0 \end{vmatrix}=$

$=1480$

podobně další

2.9.

a) $\frac{1}{2}\det A = 740$ b) $3.\det A = 4440$ c) $-\det A = -1480$

d) $-2\det A = -2960$ e) $-\frac{1}{2}\det A = -740$ f) $-\det A = -1480$

g) $4.2.(-9).5 = -360$ h) 0 i) 0

j) 0 k) 0 l) $\frac{1}{5}\cdot 2\cdot \det A = 592$

2.10. Zisťovať, či sú regulárne alebo singulárne, má zmysel o maticiach: a),e), f),g),h),i),j),k)
Regulárne: a),e),j),k) Singulárne: f),g),h),i)

a) hod = 4 b) hod = 3 c) hod = 3

d) hod = 2 e) hod = 2 f) hod = 3

g) hod = 2 h) hod = 3 i) hod = 3

j) hod = 3 k) hod = 4 l) hod = 2

2.11.

a) $\begin{pmatrix} 0 & \frac{1}{4} & 0 & \frac{1}{20} \\ \frac{9}{37} & -\frac{19}{74} & \frac{1}{37} & -\frac{3}{10} \\ \frac{1}{37} & \frac{1}{37} & -\frac{4}{37} & 0 \\ \frac{9}{37} & -\frac{19}{74} & \frac{1}{37} & -\frac{1}{10} \end{pmatrix}$ e) $\begin{pmatrix} -\frac{1}{11} & -\frac{5}{22} \\ \frac{2}{11} & -\frac{1}{22} \end{pmatrix}$

j) $\begin{pmatrix} 2 & 1 & -1 \\ 2 & -1 & 2 \\ 3 & 0 & 1 \end{pmatrix}$ k) $\begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$

2.12. a) $\left(\frac{-10}{3}, \frac{11}{3}, -3, \frac{10}{3}\right)$ b) $(0,0,0,0)$ c) neexistuje d) $(1,2,1,3)$
e) $(-18a + 1, 2a + 3, 11a - 2)$, kde $a \in R$ f) $(1,2,3)$ g) neexistuje
h) $(1 - 2a, a, 0)$, kde $a \in R$ i) $(a, b, 22a - 33b - 11, -16a + 24b + 8)$, kde $a, b \in R$
j) $(0,0,0)$ k) $(a + 2, 2 - a, a + 3)$, kde $a \in R$ l) $(3, 0, -5, 11)$
m) $(8a - 7b, -6a + 5b, a, b)$, kde $a, b \in R$ n) neexistuje