$$
\begin{pmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
\vdots & \vdots & & \vdots \\
a_{i1} & a_{i2} & \cdots & a_{in} \\
\vdots & \vdots & & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{pmatrix}_{m \times n}
\cdot
\begin{pmatrix}
b_{11} & \cdots & b_{1j} & \cdots & b_{1k} \\
b_{21} & \cdots & b_{2j} & \cdots & b_{2k} \\
\vdots & & \vdots & & \vdots \\
b_{n1} & \cdots & b_{nj} & \cdots & b_{nk}
\end{pmatrix}_{n \times k}
=
\begin{pmatrix}
c_{11} & \cdots & c_{1j} & \cdots & c_{1k} \\
\vdots & & & & \vdots \\
c_{i1} & \cdots & c_{ij} & \cdots & c_{ik} \\
\vdots & & & & \vdots \\
c_{m1} & \cdots & c_{mj} & \cdots & c_{mk}
\end{pmatrix}_{m \times k}
$$

$n \; != ! \; n$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \cdots + a_{in} b_{nj}$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1(j-1)} & a_{1j} & a_{1(j+1)} & \cdots & a_{1k} \\ \vdots & & \vdots & \vdots & \vdots & & \\ a_{(i-1)1} & \cdots & a_{(i-1)(j-1)} & a_{(i-1)j} & a_{(i-1)(j+1)} & \cdots & a_{(i-1)k} \\ a_{i1} & \cdots & a_{i(j-1)} & a_{ij} & a_{i(j+1)} & \cdots & a_{ik} \\ a_{(i+1)1} & \cdots & a_{(i+1)(j-1)} & a_{(i+1)j} & a_{(i+1)(j+1)} & \cdots & a_{(i+1)k} \\ \vdots & & \vdots & & & & \\ a_{m1} & \cdots & a_{m(j-1)} & a_{mj} & a_{m(j+1)} & \cdots & a_{mk} \end{pmatrix}_{m \times k} = A$$

$$\begin{pmatrix} a_{11} & \cdots & a_{1(j-1)} & a_{1(j+1)} & \cdots & a_{1k} \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{(i-1)1} & \cdots & a_{(i-1)(j-1)} & a_{(i-1)(j+1)} & \cdots & a_{(i-1)k} \\ a_{(i-1)1} & \cdots & a_{(i+1)(j-1)} & a_{(i+1)(j+1)} & \cdots & a_{(i+1)k} \\ \vdots & & \vdots & \vdots & & \vdots \\ a_{m1} & \cdots & a_{m(j-1)} & a_{m(j+1)} & \cdots & a_{mk} \end{pmatrix}_{(m-1) \times (k-1)} = A_{ij}$$

Príklad:

$$A = \begin{pmatrix} 1 & 1 & 2 & 4 \\ 0 & -1 & 8 & 5 \\ 0 & 0 & 0 & 3 \\ 1 & 2 & 4 & -1 \end{pmatrix}$$

Vypočítajte $A_{12} \cdot A_{34}$!

Riešenie:

$$A_{12} = \begin{pmatrix} 0 & 8 & 5 \\ 0 & 0 & 3 \\ 1 & 4 & -1 \end{pmatrix} \qquad A_{34} = \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 8 \\ 1 & 2 & 4 \end{pmatrix}$$

$$A_{12} \cdot A_{34} = \begin{pmatrix} 0 & 8 & 5 \\ 0 & 0 & 3 \\ 1 & 4 & -1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 1 & 2 \\ 0 & -1 & 8 \\ 1 & 2 & 4 \end{pmatrix} =$$

$$= \begin{pmatrix} 0.1+8.0+5.1 & 0.1+8.(-1)+5.2 & 0.2+8.8+5.4 \\ 0.1+0.0+3.1 & 0.1+0.(-1)+3.2 & 0.2+0.8+3.4 \\ 1.1+4.0+(-1).1 & 1.1+4.(-1)+(-1).2 & 1.2+4.8+(-1).4 \end{pmatrix} =$$

$$= \begin{pmatrix} 5 & 2 & 84 \\ 3 & 6 & 12 \\ 0 & -5 & 30 \end{pmatrix}$$

Sarusovo pravidlo:

$$\begin{array}{ccc|cc} a_{11} & a_{12} & a_{13} & a_{11} & a_{12} \\ a_{21} & a_{22} & a_{23} & a_{21} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{31} & a_{32} \\ - & - & + & + & - \end{array} = a_{11}a_{22}a_{33} + a_{12}a_{23}a_{31} + a_{13}a_{21}a_{32} - a_{13}a_{22}a_{31} - a_{11}a_{23}a_{32} - a_{12}a_{21}a_{33}$$

Príklad:

$$\det A = \begin{vmatrix} 1 & 0 & 3 \\ 4 & 2 & 8 \\ 0 & 0 & 1 \end{vmatrix} = ?$$

Riešenie:

a.) Sarrusovo pravidlo:

$$\det A = 1\cdot 2\cdot 1 + 0\cdot 8\cdot 0 + 3\cdot 4\cdot 0 - 3\cdot 2\cdot 0 - 8\cdot 0\cdot 1 - 1\cdot 0\cdot 4 =$$
$$= \underline{\underline{2}}$$

b.) Laplaceov rozvoj podľa 2. riadku:

$$\det A = (-1)^{2+1}\cdot 4\cdot \begin{vmatrix} 0 & 3 \\ 0 & 1 \end{vmatrix} + (-1)^{2+2}\cdot 2\cdot \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + (-1)^{2+3}\cdot 8\cdot \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} =$$
$$= (-4)\cdot 0 + 1\cdot 2 + (-1)\cdot 8\cdot 0 = \underline{\underline{2}}$$

Laplaceov rozvoj podľa 3. riadku:

$$\det A = (-1)^{3+1}\cdot 0\cdot \begin{vmatrix} 0 & 3 \\ 2 & 8 \end{vmatrix} + (-1)^{3+2}\cdot 0\cdot \begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} + (-1)^{3+3}\cdot 1\cdot \begin{vmatrix} 1 & 0 \\ 4 & 2 \end{vmatrix} = \underline{\underline{2}}$$

Laplaceov rozvoj podľa 2. stĺpca:

$$\det A = (-1)^{1+2}\cdot 0\cdot \begin{vmatrix} 4 & 8 \\ 0 & 1 \end{vmatrix} + (-1)^{2+2}\cdot 2\cdot \begin{vmatrix} 1 & 3 \\ 0 & 1 \end{vmatrix} + (-1)^{3+2}\cdot 0\cdot \begin{vmatrix} 1 & 3 \\ 4 & 8 \end{vmatrix} = \underline{\underline{2}}$$