

Limita typu: $\frac{0}{0}$

$$1: \lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow 2} \frac{2x}{1} = 2(2) = 4$$

$$2: \lim_{x \rightarrow 0} \frac{\operatorname{tg} 3x}{\sin 2x} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow 0} \frac{3 \frac{1}{\cos^2 3x}}{2 \cos 2x} = \frac{3(1)}{2(1)} = \frac{3}{2}$$

$$3: \lim_{h \rightarrow 0} \frac{\sqrt[3]{8+h} - 2}{h} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{h \rightarrow 0} \frac{\frac{1}{3}(8+h)^{-2/3}(1)}{1} = \lim_{h \rightarrow 0} \frac{1}{3(8+h)^{2/3}} = \frac{1}{3(8)^{2/3}} = \frac{1}{12}$$

$$4: \lim_{x \rightarrow \pi/3} \frac{\cos x - 1/2}{x - \pi/3} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow \pi/3} \frac{-\sin x}{1} = -\sin(\pi/3) = -\frac{\sqrt{3}}{2}$$

$$5: \lim_{x \rightarrow 0} \frac{e^x - x - 1}{x^2} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow 0} \frac{e^x - 1}{2x} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow 0} \frac{e^x}{2} = \frac{1}{2}$$

$$6: \lim_{x \rightarrow +\infty} \frac{1/x^2}{\sin(1/x)} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{x \rightarrow +\infty} \frac{-2/x^3}{\cos(1/x)(-1/x^2)} = \lim_{x \rightarrow +\infty} \frac{2/x}{\cos(1/x)} = \frac{0}{1} = 0,$$

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$$\lim_{x \rightarrow +\infty} \frac{1/x^2}{\sin(1/x)} = \lim_{y \rightarrow 0^+} \frac{y^2}{\sin y} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{y \rightarrow 0^+} \frac{2y}{\cos y} = \frac{2(0)}{1} = 0, \text{ kde } y = 1/x.$$

Limita typu: $\frac{\infty}{\infty}$

$$1: \lim_{x \rightarrow +\infty} \frac{3x^2 + 5x - 7}{2x^2 - 3x + 1} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow +\infty} \frac{6x + 5}{4x - 3} = \lim_{x \rightarrow +\infty} \frac{6}{4} = \frac{3}{2}$$

$$2: \lim_{x \rightarrow -\infty} \frac{3x - 1}{x^2 + 1} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow -\infty} \frac{3}{2x} = \frac{3}{2} \lim_{x \rightarrow -\infty} \frac{1}{x} = \frac{3}{2}(0) = 0$$

$$3: \lim_{x \rightarrow \infty} \frac{3x^3 - 4}{2x^2 + 1} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow \infty} \frac{9x^2}{4x} = \lim_{x \rightarrow \infty} \frac{18x}{4} = \infty$$

$$5: \lim_{x \rightarrow +\infty} \frac{\ln(x^2 + 1)}{\ln(x^3 + 1)} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow +\infty} \frac{\frac{2x}{x^2 + 1}}{\frac{3x^2}{x^3 + 1}} = \lim_{x \rightarrow +\infty} \frac{2x(x^3 + 1)}{3x^2(x^2 + 1)} = \lim_{x \rightarrow +\infty} \frac{2x^4 + 2x}{3x^4 + 3x^2} =$$

$$= \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow +\infty} \frac{8x^3 + 2}{12x^3 + 6x} = \lim_{x \rightarrow +\infty} \frac{24x^2}{36x^2 + 6} = \lim_{x \rightarrow +\infty} \frac{48x}{72x} = \frac{48}{72} = \frac{2}{3}$$

$$6: \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x^2} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow 0^+} \frac{1/x}{-2/x^3} = \lim_{x \rightarrow 0^+} \frac{x^3}{-2x} = \lim_{x \rightarrow 0^+} \frac{x^2}{-2} = \frac{0^2}{-2} = 0$$

Limita typu: $0 \cdot \infty$

$$1: \lim_{x \rightarrow 0^+} x \ln x = (0 \cdot \infty) = \lim_{x \rightarrow 0^+} \frac{\ln x}{1/x} = \left(\frac{\infty}{\infty} \Rightarrow L'H \right) = \lim_{x \rightarrow 0^+} \frac{1/x}{-1/x^2} = \lim_{x \rightarrow 0^+} \frac{-x^2}{x} = \lim_{x \rightarrow 0^+} (-x) = 0$$

$$2: \lim_{x \rightarrow +\infty} x \sin\left(\frac{1}{x}\right) = (0 \cdot \infty) = \lim_{x \rightarrow +\infty} \frac{\sin\left(\frac{1}{x}\right)}{1/x} = \left(\frac{0}{0} \Rightarrow L'H \right) = \lim_{y \rightarrow 0^+} \frac{\sin y}{y} = 1 \text{ [Let } y = 1/x \text{.]}$$