Starting with standard definitions of Euler's beta and gamma integrals

$$B(a,b) = \int_0^1 \mathrm{d}t t^{a-1}(1-t)^{b-1},\ \Gamma(a) = \int_0^\infty \mathrm{d}t t^{a-1}e^{-t}$$

we have for the product of two gamma-integrals (due to non-negativeness of the function to be integrated)

$$\Gamma(a)\Gamma(b) = \int_0^\infty \mathrm{d}t t^{a-1}e^{-t}\int_0^\infty \mathrm{d}s s^{b-1}e^{-s} = \int_{\mathbb{R}^+\times\mathbb{R}^+} t^{a-1}s^{b-1}e^{-(t+s)}.$$

Introducing new variables $[x,y]$:

$$t = xy,\ s = x(1-y)$$

in the last written double integral we obtain

$$\Gamma(a)\Gamma(b) = \int_0^\infty \mathrm{d}x \int_0^1 \mathrm{d}y x x^{a-1}y^{a-1}x^{b-1}(1-y)^{b-1}e^{-x} = \int_0^\infty \mathrm{d}x x^{a+b-1}e^{-x}\int_0^1 \mathrm{d}y y^{a-1}(1-y)^{b-1}.$$

In other words

$$\Gamma(a)\Gamma(b) = \Gamma(a+b)B(a,b),$$

or

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}.$$