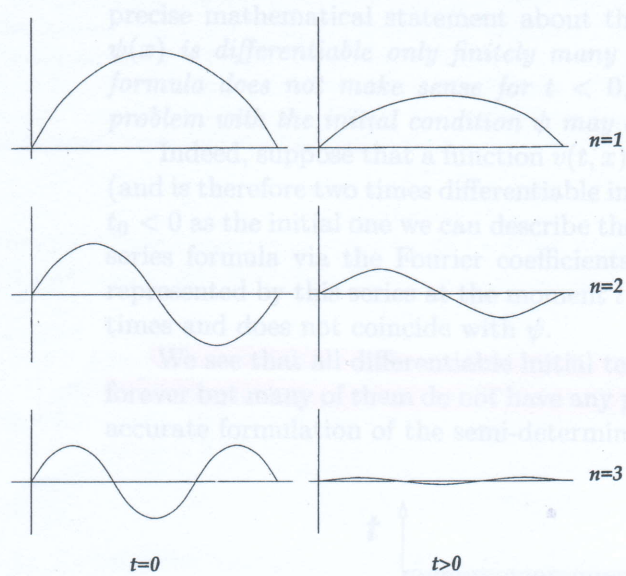
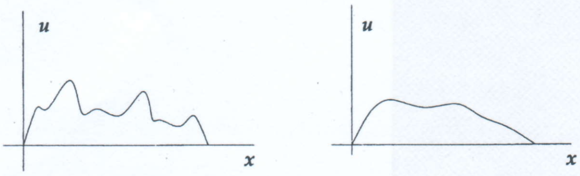
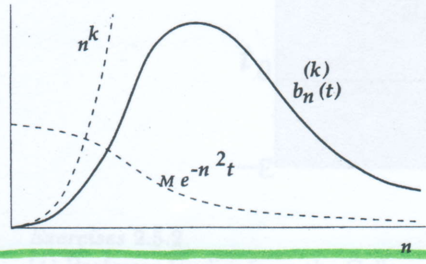
was legitimate and so the sum of the series is equal to the k-th partial derivative $\partial^k v/\partial x^k$.



On irreversibility of heat conduction





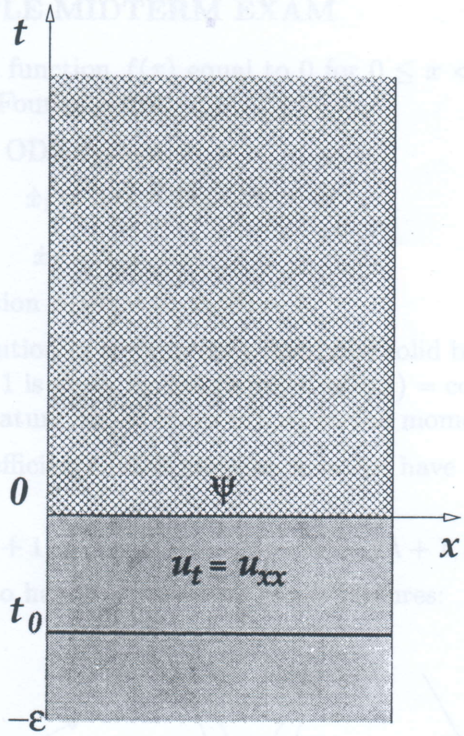
Let us now look at the heat conduction as a dynamical system where the current state is described by a current temperature distribution in the bar, and the heat equation is to describe the time evolution. The solution formula tells us how to find future states of the system via the current state. Does the current state determine the past history of the system?

We have seen examples of initial conditions with Fourier coefficients $b_n(0)$ proportional to $1/n$ or $1/n^2$. For such initial conditions the coefficients $b_n(t) = b_n(0)e^{-n^2 t}$ of the solution series grow with n when $t < 0$. Thus for $t < 0$ the series does not converge at all, and the general solution formula tells us nothing about the temperature distribution in the past. This observation agrees with the following intuitive argument: as a result of heat conduction, different initial temperature distributions eventually become homogeneous, hence the same, or almost the same. Therefore we should be unable to recover the initial distribution from a much later one. Thus heat conduction phenomena are semi-deterministic: the future is determined by the current state while, generally speaking, the past is not.

It turns out that this heuristic principle can be supported by the following precise mathematical statement about the heat equation: *if the initial condition* $\psi(x)$ *is differentiable only finitely many times, then not only the series solution formula does not make sense for* $t < 0$, *but no solution to the boundary value problem with the initial condition* ψ *may exist on any interval* $-\varepsilon < t < 0$.

Indeed, suppose that a function $v(t,x)$ satisfies the heat equation on $-\varepsilon < t \leq 0$ (and is therefore two times differentiable in x for any such t). Considering a moment $t_0 < 0$ as the initial one we can describe the function $v(t,x)$ for $t > t_0$ by the Fourier series formula via the Fourier coefficients for $v(t_0, x)$. Thus the function $v(0,x)$ represented by this series at the moment $t = 0 > t_0$ is differentiable infinitely many times and does not coincide with ψ.

We see that all differentiable initial temperature distributions are going to live forever but many of them do not have any past. This conclusion is a mathematically accurate formulation of the semi-deterministic property of heat conduction.



Exercises 2.5.2.

(a) Professor Foulier from the College of Letters and Digits invented new short-term memory hardware. In order to store a string of eight binary digits he suggested to divide a solid bar into 8 equal parts, heat each part to the temperature 1 or cool it to 0 depending on the corresponding digit and then insulate the bar. For instance, by heating the left half of the bar to 1 and cooling