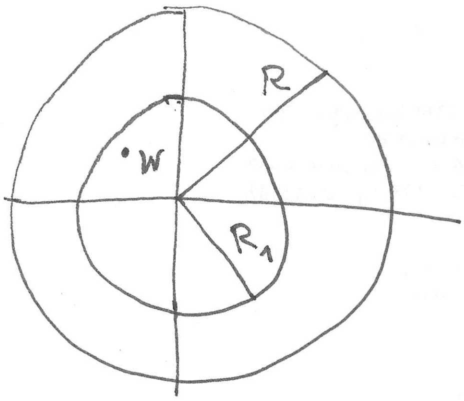
$$f(z) = \sum_n a_n z^n \quad \ldots \quad R > 0 \text{ (kladný polomer konv.)}$$

$$f_1(z) := \sum_n a_n \cdot n z^{n-1} \quad \ldots \quad \cancel{R_1} = R$$

$$f_2(z) = \sum_n a_n n(n-1) z^{n-2} \quad \ldots \quad \cancel{R_2} = R$$



$$\text{nech } \boxed{\sum_n |a_n| n(n-1) R_1^{n-2} = M}$$

$$\left| \frac{z^n - w^n}{z-w} - n w^{n-1} \right| = \left| \frac{(w+z-w)^n - w^n}{z-w} - n w^{n-1} \right| =$$

$$= \left| \sum_{i=1}^{n} \binom{n}{i} w^{n-i} (z-w)^{i-1} - n w^{n-1} \right| =$$

$$= \left| \sum_{i=2}^{n} \binom{n}{i} w^{n-i} (z-w)^{i-1} \right| =$$

$$= |z-w| \cdot \underbrace{\left| \sum_{i=2}^{n} \binom{n}{i} w^{n-i} (z-w)^{i-2} \right|}_{=G}$$

t.j.:

$$\left| \frac{z^n - w^n}{z-w} - n w^{n-1} \right| = |z-w| \cdot G$$

Teraz:

$$G = \left| \sum_{i=0}^{n-2} \binom{n}{i+2} w^{n-i-2} (z-w)^i \right|$$

a ďalej (chceme ukázať, že G nie je moc veľké !!!)

$$\binom{n}{i+2} = \frac{n(n-1)\dots(n-i-1)}{(i+2)(i+1)\dots 2\cdot 1} \leq$$

$$\leq \frac{n(n-1)}{2} \frac{(n-2)(n-3)\dots(n-2+i+1)}{i(i-1)\dots 2\cdot 1} =$$

$$= \frac{n(n-1)}{2} \cdot \binom{n-2}{i}$$

takže:

$$G \leq \frac{1}{2} n(n-1) \sum_{i=0}^{n-2} \left| \binom{n-2}{i} (z-w)^i w^{n-i-2} \right|$$

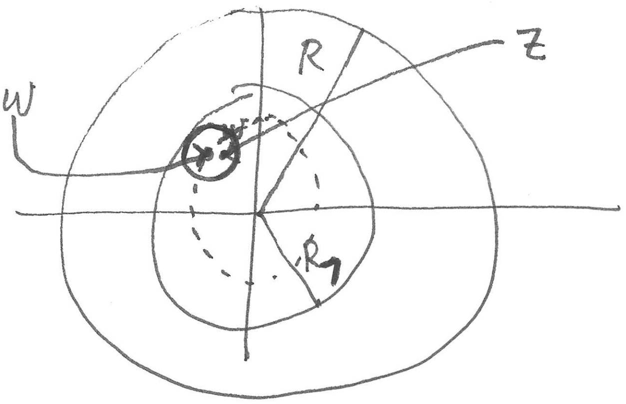
$$= \frac{1}{2} n(n-1) (|w| + |z-w|)^{n-2}$$

takže:

$$\left| \frac{z^n - w^n}{z-w} - n w^{n-1} \right| \leq \frac{1}{2} n(n-1) (|w| + |z-w|)^{n-2} \cdot |z-w|$$

Ďalej vyberieme body z a w k sebe tak (aj tak budeme skúmať $z-w \to 0$), aby:

$$|z-w| < R_1 - |w|$$



Potom:

$$\left|\frac{f(z)-f(w)}{z-w} - f_1(w)\right| =$$

$$= \left|\sum_{n=1}^{\infty} a_n \left\{\frac{z^n - w^n}{z-w} - n w^{n-1}\right\}\right| \leq$$

$$\leq \sum_{n=1}^{\infty} |a_n| \frac{(n-1)n}{2} |z-w| \left(|w| + |z-w|\right)^{n-2} \leq$$

$$\leq \frac{1}{2}|z-w| \sum_{n=1}^{\infty} n(n-1)|a_n| R_1^{n-2} =$$

$$= \frac{1}{2}|z-w| M \Rightarrow \left|\frac{f(z)-f(w)}{z-w} - f_1(w)\right| \leq$$

$$\leq \frac{1}{2} M |z-w| \xrightarrow{z \to 0} 0 \Rightarrow \boxed{f_1(w) = \frac{df}{dz}(z=w)}$$