pár riešených príkladov z nevlastných integrálov

$\boxed{1}$ nájsť D_F (obor konvergencie F):

$$F(x) = \int_1^\infty \frac{t^x}{(1+t^x)^x}\,dt \sim \int^{\infty} (\dots)$$

(i) ak $\underline{x \geq 0} \Rightarrow$

$$F \sim \int^\infty t^{x-x^2}\,dt \quad \text{konv.} \Leftrightarrow \quad x - x^2 < -1$$

$$\Leftrightarrow \quad x^2 - x - 1 > 0 \Leftrightarrow \quad x > \tfrac{1}{2}\left[1+\sqrt{5}\right] \overset{\text{ozn.}}{\equiv} x_0$$

t.j.: $(x_0, \infty) \subset D_F$

$[0, x_0] \cap D_F = \emptyset$

(ii) ak $\underline{x < 0} \Rightarrow$

$$F \sim \int^\infty t^x\,dt \quad \text{konv.} \Leftrightarrow \quad x < -1$$

záver: $\boxed{D_F = (-\infty, -1) \cup (x_0, +\infty)}$

$\boxed{2}$ nájsť D_F:

$$F(x) = \int_0^\infty t^x \cos t^3 \, dt$$

najprv $\int_0^\infty (\dots)$

$x \geq 0$: $\sim \int^{\infty} (\dots)$

$x < 0$: $\sim \int^{\infty} (\dots) + \int_{0} (\dots)$

(i) $x \geq 0$: $\int (\dots) = \left| t^3 = u \right| = \int^\infty u^{\frac{x}{3}} \cos(u) \frac{1}{3} u^{-\frac{2}{3}} du$

$$= \frac{1}{3} \underbrace{\int^\infty u^{\frac{x-2}{3}} \cos u \, du}_{J}$$

teraz pre $x < 2$ je

$u^{\frac{x-2}{3}} \searrow 0$ pre $u \to \infty$

$\int \cos u \, du$ je ohraničený.

$\Rightarrow \boxed{[0,2) \subset D_F}$

ďalej pre $x \geq 2$: J nesplňa C-B podmienku (vezmi napr. integrály $\int_{2k\pi}^{2k\pi + \frac{\pi}{2}} (\dots)$) $\Rightarrow$

$[2, \infty) \cap D_F = \emptyset$

Záver:

$D_F = (-1, 2)$

(ii) $x < 0$:

(ii A) $\int^\infty (\dots) \sim J$ a ten konverguje

(ii B) $\int_0 (\dots) \sim \int_0 t^x dt$ konv. $\iff \boxed{x > -1}$

$\boxed{3}$ či konverguje integrál:

$$J = \int^{\infty} \frac{dt}{(\ln t)^{\ln t}}$$

$$J = \left| \ln t = u \right| = \int^{\infty} \frac{e^u\, du}{u^u} = \int^{\infty} e^u e^{-u \ln u}\, du$$

$$\overset{\varepsilon > 0}{<} \int^{\infty} e^{-\varepsilon \cdot u}\, du < +\infty \Rightarrow \underline{J \text{ konverguje}}$$

$\boxed{4}$ či konverguje

$$J = \int_0^{\infty} \frac{dx}{(1 - x - x^3 - x^5)^{1/3}}$$

f-cia $\mathbb{R}_0^+ \ni x \longmapsto 1 - x - x^3 - x^5$ je monotónna začína s hodn. 1 a má limitu v $+\infty$ rovnú $-\infty$ preto existuje jediný a jednoduchý nulový bod tejto f-cie, nech je to x_0 (>0).

Potom:

$$J \sim \int^{x_0} (\dots) + \int_{x_0}^{\infty} (\dots)$$

$$\left.\begin{aligned} &(i) \int^{x_0} (\dots) \sim \int^{x_0} |x - x_0|^{-1/3} dx < +\infty \\ &(ii) \int^{\infty} (\dots) \sim \int^{\infty} x^{-5/3} dx < +\infty \end{aligned}\right\} \Rightarrow \underline{J \text{ konverguje}}$$

$\boxed{5}$ či konverguje

$$J = \int_0^\infty \frac{1}{\sqrt{x}} \sin\frac{1}{x}\, dx$$

$$J \sim \int_0 (\dots) + \int^\infty (\dots)$$

- $\int^\infty (\dots) \sim \int^\infty x^{-3/2}\, dx < +\infty$
- $\int_0 |(\dots)| \overset{(*)}{\leq} \int_0 \frac{1}{\sqrt{x}}\, dx < +\infty$

$\Rightarrow$ J konverguje

$\boxed{6}$ či konverguje

$$J = \int_0^\infty \frac{1}{x} \sin\frac{1}{x}\, dx$$

podobne ako v $\boxed{5}$ $J \sim \int^\infty (\dots) + \int_0 (\dots)$

pre $\int^\infty (\dots) \sim \int^\infty x^{-2}\, dx < +\infty$

ale odhad typu $(*)$ už nič nepomôže

lebo $\int_0 |(\dots)| \leq \int_0 \frac{1}{x}\, dx = +\infty$

inak: $J_0 = \int_0^\varepsilon \frac{1}{x} \sin\frac{1}{x}\, dx = \left| \frac{1}{x} = y \right| = \int_{1/\varepsilon}^\infty y \sin y \frac{1}{y^2}\, dy$

$$= \int_{1/\varepsilon}^\infty \frac{\sin y}{y}\, dy, \quad \left\langle \begin{array}{l} \int \sin y \text{ je ohr.} \\ \frac{1}{y} \searrow 0 \ (y \to +\infty) \end{array} \right\} \Rightarrow J_0 \text{ konverguje}$$

$\Rightarrow$ $\boxed{J \text{ konverguje}}$

7 najít D_F :

$$F(a) = \int_0^\infty \frac{dx}{a + x^a}$$

(i) $\underline{a>0}$: $F \sim \int^\infty x^{-a}\, dx$ konv $\Leftrightarrow$ $a>1$

t.j. $\begin{cases} (1,\infty) \subset D_F \\ (0,1] \cap D_F = \emptyset \end{cases}$; bokem: $0 \notin D_F$

(ii) $\underline{a<0}$ problém v $x_0 = (-a)^{1/a}$

$$F \sim \int_{x_0} (\dots) + \int^\infty (\dots)$$

$\int^\infty (\dots) \sim \int^\infty 1$ div. $\Rightarrow (-\infty, 0) \cap D_F = \emptyset$

závěr: $\boxed{D_F = (1,\infty)}$

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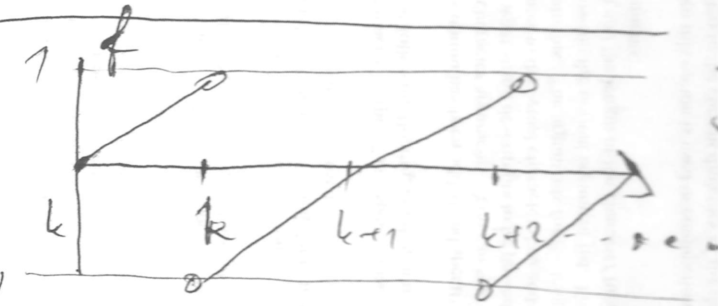

$$J = \int^\infty f(x)\, \frac{1}{x}\, dx$$

Z obrázku je: $\mathbb{R}^+ \ni A \mapsto \int^A f(x)\,dx$ je ohraničená

a dále: $\frac{1}{x} \searrow 0$ $(x \to \infty)$ proto $\underline{J \text{ konverguje}}$.

$\boxed{9}$ Nakreslite graf f-cie

$$F(a) = \int_0^\infty \frac{\sin ax}{x}\,dx$$

Zrejme $0 \in D_F$ a pre $a \neq 0$ je

$x \mapsto \int \sin ax\,dx$ ohr. f-cia a $\frac{1}{x} \overset{x\to\infty}{\searrow} 0$

preto $D_F = \mathbb{R}$ a ďalej F je <u>nepárna</u>

Ďalej $F(a) \overset{a>0}{=} \left| ax = y \right| = \int_0^\infty \frac{\sin y}{y}\,dy$

$\Rightarrow F(a) \overset{a>0}{=} \text{const} = -F(-a)$

ďalej const > 0 lebo

$$\text{const} = \int_0^\infty \frac{\sin x}{x}\,dx = \sum_{k=0}^\infty \int_{2\pi k}^{2\pi(k+1)} \frac{\sin x}{x}\,dx =$$

$$= D_0 + D_1 + D_2 + \dots$$

kde $|D_0| > |D_1| > |D_2| > \dots$

$D_{2l} > 0$; $D_{2l+1} < 0$

<u>preto</u>: