pár riešených úloh na Frullaniho vzorce

1 $I(a)=\int_0^\infty \frac{\sin(ax)-\sin x}{x}\,dx \quad ; \quad a>0$

$\int^\infty \frac{\sin x}{x}\,dx$ konverguje preto:

$$\boxed{I(a)=0}$$

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2 $I(a)=\int_0^\infty \frac{\cos(ax)-\cos x}{x}\,dx$, zistiť D_I ...

inak $\int^\infty \frac{\cos x}{x}\,dx$ konv. a $\cos: \mathbb{R}_0^+ \to \mathbb{R}$ je

spojitá a je párna preto $(\mathbb{R}\setminus\{0\}) \subset D_I$

a platí:

$\underline{a\neq 0}$: $\int_0^\infty \frac{\cos ax - \cos x}{x}\,dx = \cos(0)\cdot \ln\frac{1}{a} =$

$= -\ln a$

ak $a=0$ ... $\int_0^\infty \frac{1-\cos x}{x}\,dx \sim \int^\infty \frac{1-\cos x}{x}\,dx \neq$

$\neq$ ~~...~~ $\int^\infty \frac{dx}{x}dx - \underbrace{\int^\infty \frac{\cos x}{x}\,dx}_{K} \Rightarrow 0 \notin D_I$

$\quad D$

finálne:

$$\boxed{D_I=\mathbb{R}^+_- \wedge I(a)=-\ln a}$$

$\boxed{3}$ $$\int_0^\infty \frac{e^{-x^2} - e^{-ax^2}}{x}\,dx = \quad \boxed{a>0} \quad \int_0^\infty \frac{e^{-x^2} - e^{-(\sqrt{a}\,x)^2}}{x}\,dx =$$

$$= e^{0^2} \cdot \ln \frac{\sqrt{a}}{1} = \frac{1}{2} \ln a$$

$\boxed{4}$ $$I(a) = \int_0^\infty \frac{\ln(1+x^2) - \ln(1+ax^2)}{x}\,dx \; ; \; a > 0$$

v tomto prípade zrejme $I(1) = 0$ <u>ale</u> pre $a \neq 1$:

$$\ln(1+x^2) - \ln(1+ax^2) = \ln \frac{1+x^2}{1+ax^2} \underset{x\to\infty}{\sim} -\ln a$$

preto $$\overset{a\neq 1}{I(a)} \sim \int^\infty \frac{dx}{x} = +\infty$$

preto $D_I = \{1\}$ a $I(1) = 0$

$\boxed{5}$ $$\int_0^\infty \frac{\cos\sqrt{x} - \cos 2\sqrt{x}}{x}\,dx$$

otázka je či je $\int \frac{\cos\sqrt{x}}{x}\,dx$ zač... $= \left| x = y^2 \right| =$

$$= \int^\infty \frac{\cos y}{y^2}\, 2y\, dy \sim \int^\infty \frac{\cos y}{y}\, dy$$ <u>konv.</u> $\Rightarrow$ platí:

$$\int_0^\infty \frac{\cos\sqrt{x} - \cos 2\sqrt{x}}{x}\,dx = \cos 0 \cdot \ln\frac{4}{1} = \underline{\underline{\ln 4}}$$